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MULTIPLE-DECISION SELECTION
AND RANKING PROCEDURES

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FINAL REPORT ON CONTRACT DAAG29-73-C-0008

by

Robert E. Bechhofer

November 12, 1976

U.S. ARMY RESEARCH OFFICE

CONTRACT DAAG29-73-C-0008 *new*

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FINAL REPORT ON CONTRACT DAA39-73-2-9002

Robert E. Macomber

November 12, 1978

U.S. ARMY RESEARCH OFFICE

CONTRACT DAA39-73-2-9002

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FINAL REPORT ON CONTRACT DAAG29-73-C-0008

This is the final report on Contract DAAG29-73-0008 entitled "Multiple-Decision Selection and Ranking Procedures," and covers the period July 1, 1973-September 30, 1976. The present contract is a follow-up to Contract DA-31-124-ARO-D-474 which had a similar mission and covered the period September 15, 1966-June 30, 1973. Because of the close relationship between the two contracts, the present report lists some papers, the research for which was initiated or completed under the earlier contract but which were not published in final form until after the final report for the earlier contract was submitted. In addition it lists all Ph.D. dissertations and M.S. theses the research for which was supported in whole or in part by either contract. The complete contents of the present report are described below.

The following pages list the technical reports and published papers (and those accepted for or submitted for publication) which were written with the full or partial support of the contract. The contents of most of these reports and papers have already been described in the semi-annual progress reports; abstracts of the contents of the four newest ones are given herein. Also included is a list of the scientific personnel supported by the contracts, and the advanced degrees earned by them while supported by the contracts.

Most of the papers deal with research on the subject of "Statistical Multiple Decision Ranking Procedures," a statistical methodology pioneered by the Principal Investigator and several colleagues; some of the papers deal with related statistical techniques, and a few with probabilistic developments. An overview of the ranking and selection approach (including many important references) is contained in the expository paper entitled "Ranking and Selection Procedures" which the Principal Investigator delivered at the Twentieth Conference on the Design of Experiments in Army Research Development

and Testing, October 1974. The importance of studying statistical problems from this point of view is now generally recognized, and several hundred research papers have now been written on the subject by many investigators. As is usually the case with new research developments, it takes some time before their implications are understood and appreciated by practitioners; it is anticipated that this process will be accelerated for ranking and selection procedures as a consequence of new books and expository papers now being written on the subject.

Some of the many results obtained on this contract were reported on at the meetings of various professional societies or groups; in the future special efforts will be made to make a larger body of practitioners aware of them. The Principal Investigator has been a regular participant in the Annual Design of Experiments Conferences sponsored by ARO-D, and these conferences have several times provided a forum for such presentations.

The mission of the present contract is being continued under a new contract DAAG29-77-C-0003. The Principal Investigator is deeply indebted to the Army Research Office-Durham for its strong support of this research activity.

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Summary of Research Completed on the Contract

Technical reports, the research for which was supported in whole or in part by ARO Contract DAAG29-73-C-0008.

Bechhofer, R. E. and Tamhane, A. C.: An iterated integral representation for a multivariate normal integral having block covariance structure, TR 211, January 1974.

Farquhar, P. H.: Fractional hypercube decompositions of multiattribute utility functions, TR 222, August 1974.

Bechhofer, R. E.: A two-sample procedure for selecting the population with the largest mean from several normal populations with unknown variances: some comments on Ofosu's paper, TR 233, October 1974.

Bechhofer, R. E. and Turnbull, B. W.: A $(k+1)$ -decision single-stage selection procedure for comparing k normal means with a fixed known standard: the case of common known variance, TR 242, December 1974.

Kreimerman, J.: A bivariate test of goodness of fit based on a gradually increasing number of order statistics, TR 250, June 1975.

Awate, P.: Dynamic programming with negative rewards and average reward criterion, TR 251, May 1975.

Bechhofer, R. E. and Turnbull, B. W.: A $(k+1)$ -decision two-stage selection procedure for comparing k normal means with a fixed known standard: the case of common unknown variances, TR 256, May 1975.

Turnbull, B. W.: Multiple decision rules for comparing several populations with a fixed known standard, TR 257, June 1975.

Tamhane, A.: On minimax multistage elimination type rules for selecting the largest normal mean, TR 259, May 1975.

Tamhane, A.: A minimax two-stage permanent elimination type procedure for selecting the smallest normal variance, TR 260, June 1975.

Gelber, R.: A sequential goodness-of-fit test for composite hypotheses involving unknown scale and location parameters, TR 266, August 1975.

Bechhofer, R. E., Santner, T. J., and Turnbull, B. W.: An application of majorization to the problem of selecting the largest interaction in a two-factor experiment, TR 292, May 1976.

Abstract: The problem of devising a single-stage procedure for the goal of selecting the factor-level combination associated with the largest positive interaction is studied for the usual linear model underlying a 2-factor $r \times c$ ($r \geq 2$, $c \geq 3$) experiment involving qualitative variables when the common variance is known. The formulation adopted for this goal

employs a generalized indifference-zone approach which requires the experimenter to specify quantities $\{\Delta^*, \delta^*, P^*\}$

$(0 < \Delta^* < \infty, 0 < \delta^* < \frac{(r-1)(c-1) - 1}{(r-1)(c-1)} \Delta^*, \frac{1}{rc} < P^* < 1)$ prior to the start of experimentation. These quantities are then incorporated into a probability requirement which must be guaranteed by any potential selection procedure. A single-stage procedure is proposed to accomplish this objective. In order to implement the procedure it is necessary to determine the minimum sample size which will guarantee the probability requirement.

As a first step in this determination, an exact expression for the probability of a correct selection (PCS) using the procedure is derived for the $r \times c$ case. This PCS must be minimized with respect to the interactions γ_{ij} ($1 \leq i \leq r, 1 \leq j \leq c$), subject to guaranteeing the probability requirement, in order to determine the desired minimum sample size. The γ_{ij} at which this minimum PCS is achieved is called a least favorable (LF) configuration. The main result of the present paper is a theorem which gives this LF-configuration explicitly for the $2 \times c$ case; the principal tool used in the proof of the theorem is the theory of Schur-concavity and majorization in multivariate distributions as described by Marshall and Olkin. Various generalizations are proposed.

Turnbull, B. W.: The empirical distribution function with arbitrarily grouped, censored, and truncated data, TR 305, June 1976.

Abstract: This paper is concerned with the nonparametric estimation of a distribution function F , when the data are incomplete due to grouping, censoring and/or truncation. Subsets B_1, B_2, \dots, B_N of the real line are given and there are N independent observations X_1, X_2, \dots, X_N , where X_i is drawn from the truncated distribution $F(x; B_i) = P(X \leq x | X \in B_i)$, $x \in B_i$. However X_i may not be observed exactly and is known only to lie in the set $A_i \subseteq B_i$. The situation occurs frequently in survivorship, reliability, and recidivism analysis. Using the idea of self-consistency, a simple algorithm is constructed and shown to converge monotonically to yield a maximum likelihood estimate of F . The procedure compares favourably with the more cumbersome Newton-Raphson method. A test is proposed for comparing two distributions when data on one or both is incomplete and some other applications of the empirical distribution function are indicated.

Turnbull, B. W. and Weiss, L.: A likelihood ratio statistic for testing goodness of fit with randomly censored data, TR 307, August 1976.

Abstract: A likelihood ratio statistic is proposed for testing goodness of fit with grouped data which are subject to random right censoring. It is shown that, under appropriate conditions, this statistic has an asymptotic chi-square distribution which is non-central under contiguous alternatives. Some examples are given including one on marijuana usage which needs an extension of the test to the doubly censored case.

Jakobovits, R. H.: Goodness of fit tests for composite hypotheses based on an increasing number of order statistics, TR 310, September 1976.

Abstract: This paper is concerned with sequences of tests of the composite null hypothesis that the distribution function of an observed sample is a member of a specified scale-location parameter family, where the scale and location parameters are unknown and unspecified. First a test of this hypothesis proposed by L. Weiss is studied. Then several tests for the most common special case, that of normality, are proposed and analyzed.

All of the results are asymptotic, as the sample size n tends to infinity, and all of the test criteria which we examine are based on functions of an increasing subset of the ordered sample values. By increasing subset, is meant that as the sample size tends to infinity, the number of order statistics which are considered also tends to infinity. By assuming certain relationships among the number of sample quantiles in the selected subset, the separation between those quantiles, the rate at which the quantiles move into the tails of the distribution, and the smoothness and size of the tails of the density function of a standard representative of the hypothesized scale-location parameter family, it is possible to assert that the selected subset of sample quantiles is asymptotically jointly normally distributed with known mean vector and covariance matrix.

The Weiss procedure is based on a quadratic form of the sample quantiles which has an asymptotic chi-square distribution. The asymptotic power of his test is examined under sequences of contiguous alternatives of the form $H_n(x) = G((x - \theta_1)/\theta_2) + \epsilon_n(x)$, where $G(y)$ is the standard representative of the scale-location parameter family appearing in the null hypothesis, and the "disturbance functions" $\{\epsilon_n(x)\}$ satisfy certain regularity conditions. By letting the sequence $\{\epsilon_n(x)\}$ approach zero

at a certain rate and with respect to a certain measure of distance, a non-trivial power for the test is obtained.

Turning to the composite hypothesis of normality and using the same approach of selecting a gradually increasing subset of order statistics, large-sample analogs of the Wilk-Shapiro and Shapiro-Francia tests are developed. (The W-S and S-F tests are two of the most effective (small samples) but also least understood tests for normality.) It is shown that the analog of the Wilk-Shapiro statistic is asymptotically normally distributed under the null hypothesis as the sample size tends to infinity. It is also shown that, asymptotically, the analog of the Shapiro-Francia statistic, suitably standardized, has the same distribution under the null hypothesis as a certain weighted sum of independent chi-square random variables. Up to a shift in location, the latter distribution is the same as that derived by de Wet and Venter for an analog of the Shapiro-Francia test based on the complete set of order statistics. It is proved that the tests based on the above statistics are consistent. Finally, the behavior of those tests under the sequence of alternatives $\{H_n(x)\}$ is studied. It is found that the measures of distance for the tests are quite complicated. It is also found that, contrary to most small-sample empirical studies, the analog of the Shapiro-Francia test has better asymptotic power than the analog of the Wilk-Shapiro test, and it is shown by example that the analog of the Wilk-Shapiro test can be biased, even against sequences of contiguous alternatives. As a consequence of the power studies, two further statistics (modifications of those discussed above) which yield improved tests for normality are proposed and analyzed.

Research supported in whole or in part by ARO contract DAAG29-73-C-0008 or its predecessor ARO contract DA-31-124-ARO-D-474

Papers (published or accepted for publication):

Bawa, V.S.: "Asymptotic efficiency of one R-factor experiment relative to R one-factor experiments for selecting the best normal population," Journal of the American Statistical Association, Vol. 67 (1972), pp. 660-661.

Bechhofer, R.E.: "Ranking and selection procedures," Proceedings of the Twentieth Conference on the Design of Experiments in Army Research, Development, and Testing held at Fort Belvoir, Virginia, October 23-25, 1974, pp. 929-949.

Bechhofer, R.E.: "A two-sample procedure for selecting the population with the largest mean from several normal populations with unknown variances: some comments on Ofosu's paper." The comments were accepted by Ofosu, and mentioned by him in a Correction Note in Biometrika, Vol. 62, No. 1, 1975, p. 221.

Bechhofer, R.E., Santner, T.J., and Turnbull, B.W.: "Selecting the largest interaction in a two-factor experiment." To appear in the Proceedings of the Second Symposium on Statistical Decision Theory and Related Topics held at Purdue University, May 17-19, 1976.

Bechhofer, R.E. and Tamhane, A.C.: "An iterated integral representation for a multivariate normal integral having block covariance structure," Biometrika, Vol. 61, No. 3, December 1974, pp. 615-619.

Farquhar, P.H.: "A fractional hypercube decomposition theorem for multiattribute utility functions," Journal of the Operations Research Society of America, Vol. 23 (1975), pp. 941-967.

Fushimi, M.: "An improved version of a Sobel-Weiss play-the-winner procedure for selecting the better of two Bernoulli populations," Biometrika, Vol. 60, No. 3, December 1973, pp. 517-523.

Hooke, J.: "On some limit theorems for the GI/G/1 queue," Journal of Applied Probability, Vol. 7 (1970), pp. 634-640.

Hooke, J.: "A priority queue with low priority arrivals general," Journal of the Operations Research Society of America, Vol. 20 (1972), pp. 373-380.

Hooke, J.: "Some heavy traffic limit theorems for a priority queue with general arrivals," Journal of the Operations Research Society of America, Vol. 20 (1972), pp. 381-388.

Ramberg, J.S.: "Selection sample size approximations," Annals of Mathematical Statistics, Vol. 43 (1972), pp. 1977-1980.

Turnbull, B.W.: "Chebyshev-like inequalities for dam models," Journal of Applied Probability, Vol. 9 (1972), pp. 617-629.

Turnbull, B.W.: "Inequalities for branching processes," The Annals of Probability, Vol. 1 (1973), pp. 457-474.

Turnbull, B.W.: "Inequalities for multitype branching processes," The Annals of Probability, Vol. 1 (1973), pp. 475-479.

Turnbull, B.W.: "The empirical distribution function with arbitrarily grouped, censored, and truncated data." To appear in the Journal of the Royal Statistical Society, B, Vol. 38, No. 3.

Turnbull, B.W.: "Multiple decision rules for comparing several populations with a fixed known standard." To appear in Communications in Statistics.

Papers submitted for publication and currently being revised at the request of the editor:

Bechhofer, R.E. and Turnbull, B.W.: "A $(k+1)$ -decision single-stage selection procedure for comparing k normal means with a fixed known standard: the case of common known variance," and "A $(k+1)$ -decision two-stage selection procedure for comparing k normal means with a fixed known standard: the case of common unknown variance." These papers have been combined at the request of the editor, and the revised paper has been resubmitted.

Papers submitted for publication:

Frischtak, R.: "Selection of subclasses of variates based on a measure of association."

Frischtak, R.: "Selection of smallest vector coefficient of alienation: asymptotic case."

Fushimi, M.: "A non-symmetric sequential procedure for selecting the better of two binomial populations."

Tamhane, A.C.: "A minimax two-stage permanent elimination type procedure for selecting the smallest normal variance."

Tamhane, A.C.: "On 2- and 3-stage screening procedures for selecting the population having the largest mean from k normal populations with a common unknown variance."

Turnbull, B.W. and Weiss, L.: "A likelihood ratio statistic for testing goodness of fit with randomly censored data."

Graduate students supported by Contract DAAG29-73-C-0008

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- Awate, Prakash: (Ph.D. dissertation) "Dynamic programming with negative rewards and average reward criterion." September 1975, (TR 251).
- Bawa, Vijay S.: (Ph.D. dissertation) "Asymptotically optimal ranking and selection procedures." June 1970, (TR 102).
- Farquhar, Peter H.: (Ph.D. dissertation) "Fractional hypercube decompositions of multiattribute utility functions." August 1974, (TR 222).
- Frischtak, Ricardo M.: (Ph.D. dissertation) "Statistical multiple-decision procedures for some multivariate selection problems." August 1973, (TR 187).
- Gelber, Richard D.: (Ph.D. dissertation) "A sequential goodness-of-fit test for composite hypotheses involving unknown scale and location parameters." September 1975, (TR 266).
- Higgins, James E.: (M.S. thesis) "Bernoulli sampling plans which approximately minimize the maximum expected sample size." February 1969, (TR 65).
- Hooke, John: (Ph.D. dissertation) "Some limit theorems for priority queues." January 1970, (TR 91).
- Jakobovits, Ray H.: (Ph.D. dissertation) "Goodness of fit tests for composite hypotheses based on an increasing number of order statistics." September 1976, (TR 310).
- Kakumanu, Prasadara V.: (Ph.D. dissertation) "Continuous time Markov decision models with applications to optimization problems." September 1969, (TR 63).
- Kreimerman, Jose: (Ph.D. dissertation) "A bivariate test of goodness of fit based on a gradually increasing number of order statistics." June 1975, (TR 250).
- Marathe, Vijay: (Ph.D. dissertation) "Priority queuing systems with simultaneous server requirements." May 1972.
- Nocturne, Dominique: (Ph.D. dissertation) "Asymptotic efficiency of the maximum likelihood estimators for the parameters of certain stochastic processes." June 1970, (TR 105).
- Ramberg, John S.: (Ph.D. dissertation) "A multiple-decision approach to the selection of the best set of predictor variates." February 1969, (TR 79).
- Resnick, Mrs. Yvonne (Kramer): (M.S. thesis) "Hedging procedure for estimation of $(k+1)$ quantiles for a population with unknown mean." September 1969.
- Tamhane, Ajit C.: (Ph.D. dissertation): "On minimax multistage elimination type rules for selecting the largest normal mean." June 1975, (TR 259).
- Turnbull, Bruce W.: (Ph.D. dissertation) "Bounds and optimal strategies for stochastic systems." September 1971, (TR 134).

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